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P Wave Meson Spectrum in a Relativistic Model with Instanton Induced Interaction

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On the basis of the phenomenological relativistic harmonic models for quarks we have obtained the masses of P wave mesons. The full Hamiltonian used in the investigation has Lorentz scalar + vector confinement potential, along with one gluon exchange potential (OGEP) and the instanton-induced quark-antiquark interaction (III). A good agreement is obtained with the experimental masses. The respective role of III and OGEP for the determination of the meson masses is discussed.

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I. INTRODUCTION

There is a wealth of experimental data in hadron spectroscopy that would constitute a good testing ground for nonperturbative quantum chromodynamics (QCD). Since the exact form of confinement from QCD is not known, one has to go for phenomenological models. The phenomenological models are either non-relativistic quark models (NRQM) [1, 2, 3, 4, 5, 6, 7] with a suitably chosen potential, or relativistic models where the interaction is treated perturbatively. NRQM have proved to be quite successful in describing the hadronic properties. The Hamiltonian of these quark models usually contains three main ingredients: the kinetic energy, the confinement potential and an hyperfine interaction term, which has often been taken as an effective one-gluon-exchange potential (OGEP) [8]. Other types of hyperfine interaction have been introduced in the literature; from the non-relativistic reduction of the t'Hooft interaction [9, 10], termed as Instanton-Induced Interaction (III), which has already been successfully applied in several studies of the hadron spectra [7, 11, 12, 13, 14, 15, 16]. The main achievement of the III in hadron spectroscopy is the resolution of the $U_A(1)$ problem, which leads to a good prediction of the masses of η and η' mesons. The Goldstone-Boson-Exchange interaction introduced by Glozman and Riska [17] furnishes another example of hyperfine interaction; it allows a good description of the baryon spectrum and gives a correct ordering for the positive and negative parity states. The model of Glozman and Riska has however the major caveat to apply only to baryons and is thus not able to give an unified description of the spectrum of hadrons.

The successes of the NRQM in describing the spectrum of light hadrons are somehow paradoxical, as light quarks should in principle not obey a non-relativistic dynamics. This paradox has been avoided in many works based on the constituent quark model by using for the kinetic energy term of the hamiltonian a semi-relativistic or relativistic expression (see, for example,[15, 18, 19]).

In literature there are models which have tried to explain hadron spectroscopy only with OGEP [1, 4] and some models with only III [7, 15], ignoring completely the OGEP. We feel that it may be an exaggeration to eliminate OGEP completely for light quarks. The OGEP still has to be present but with a smaller strength consistent with the asymptotic freedom, since the III has to vanish for heavy quarks.

In our present work, we have made use of the relativistic harmonic model (RHM) [20]. The RHM combined with OGEP has already been used to calculate light hadrons masses, baryons magnetic moments, leptonic decay widths and N-N scattering phase shifts [21, 22, 23]. Previously, by employing the RHM [20, 24] and the NRQM [25] along with III, the ground state masses of pseudoscalar and vector mesons were investigated. In both the cases the results showed that the inclusion of III diminished the relative importance of OGEP for the hyperfine splitting. One of the aims of the investigation was also to test whether quark gluon coupling constant(α_s) can be treated as a perturbative effect and to understand the role played by the III in meson spectra [24].

In view of the apparent success of RHM [24] in the description of S-wave spectra of mesons, we feel it is worthwhile to apply it to the case of orbitally excited states. In our present work the full hamiltonian used has a Lorentz scalar plus vector confinement potential, along with central and non-central (spin-orbit and ten-

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sor) terms of OGEP and III. In addition, III also has antisymmetric spin-orbit term proportional to $\vec{L}\cdot\vec{\triangle}$ where $\vec{\triangle}$ is defined in terms of the Pauli matrices as $\frac{1}{2}(\vec{\sigma}_1-\vec{\sigma}_2)$. The full discussion of the hamiltonian is given in section II. We also discuss the parameters involved in our model in section III. The results of the calculation are presented in section IV and the conclusions are given in section V.

In our present work, we have computed the masses of the singlet and triplet light P wave mesons by including the III as a short-range nonperturbative gluon effect in addition to the perturbative conventional OGEP derived from QCD. This will allow much better understanding of the P-wave meson spectroscopy, where some of the $q\bar{q}$ quark model assignments of the known mesons are still controversial. We hope it will also allow us a better understanding of the production properties of the P-wave mesons. In literature there are numerous attempts to understand the P-wave meson spectroscopy. The reference can be found in the review [26]. One of the aims of this study is to determine explicitly the role played by instantons in meson spectra, when used in the framework of the RHM and to compare the effects of III to that of OGEP.

II. THE RELATIVISTIC HARMONIC MODEL

In the RHM [20], quarks in a hadron are confined through the action of a Lorentz scalar plus a vector harmonic oscillator potential

$$V_{\rm conf}(r)=\frac{1}{2}(1+\gamma_0)\ A^2r^2+M \tag{1}$$

where γ_0 is the Dirac matrix

$$\gamma_0=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{2}$$

M is the quark mass parameter and A^2 the confinement strength. They have a different value for each quark flavour. In the RHM, the confined single quark wave function Ψ is given by

$$\Psi=N\begin{pmatrix}\Phi\\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{P}}{E+M}\ \Phi\end{pmatrix} \tag{3}$$

with the normalization

$$N=\sqrt{\frac{2(E+M)}{3E+M}}, \tag{4}$$

E is an eigenvalue of the single particle Dirac equation with the interaction potential given by (1). We can perform a unitary transformation to eliminate the lower component of Ψ such that

$$U\ \Psi=\begin{pmatrix}\Phi\\ 0\end{pmatrix} \tag{5}$$

where U is given by

$$U=\frac{1}{N\ \left[1+\frac{P^2}{(E+M)^2}\right]}\begin{pmatrix}1 & \frac{\sigma.\mathbf{P}}{E+M}\\ -\frac{\sigma.\mathbf{P}}{E+M} & 1\end{pmatrix} \tag{6}$$

Here U is a momentum and eigenvalue E dependent transformation operator. With this transformation, the upper component Φ satisfies the equation

$$\left[\frac{P^2}{E+M}+A^2r^2\right]\ \Phi=(E-M)\ \Phi \tag{7}$$

which is like the three dimensional harmonic oscillator equation with an energy dependent parameter Ω_n^2

$$\Omega_n^2=A\ (E_n+M)^{\frac{1}{2}} \tag{8}$$

The eigenvalue of (7) is thus given by

$$E_n^2=M^2+(2n+1)\ \Omega_n^2 \tag{9}$$

Note that Eq.(7) can also be derived by eliminating the lower component of the wave function, using a Foldy-Wouthuysen transformation, as it has been done in [20].

The total energy of the hadron is obtained by adding the individual contributions of the quarks. The spurious center of mass (CM) is corrected [27] by using intrinsic operators for the $\sum_i r_i^2$ and $\sum_i \nabla_i^2$ terms appearing in the Hamiltonian. This amounts to just subtracting the CM motion zero contribution from the E^2 expression.

We come now to the description of the quark-antiquark potential; it is given by the sum of OGEP and III potential

$$V_q(r_{ij}\ )=V_{\rm OGEP}(r_{ij})+V_{\rm III}(r_{ij}), \tag{10}$$

with r_{ij} the inter-quark distance. Among the several versions of the OGEP, we have used the following one, first derived in [8] from the QCD Lagrangian in the non-relativistic limit

$$\begin{aligned}V_{\rm OGEP}(r_{ij})\ &=\ \frac{\alpha_s}{4}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j \qquad (11)\\ &\times\left[\frac{1}{r_{ij}}-\frac{\pi}{M_iM_j}\ (1+\frac{2}{3}\ \sigma_i\cdot\sigma_j)\ \delta(r_{ij})\right]\end{aligned}$$

where the first term is the residual Coulomb energy and the second-term the chromo-magnetic interaction leading to the hyperfine splittings. Here, λ_i is the generator of the color SU(3) group for the $i^{\rm th}$ quark, σ_i is the Pauli spin operator.

To model the III, we have used the form given in [7, 15]:

$$V_{\rm III}(r_{ij})=\begin{cases}-8g\delta(r_{ij})\delta_{S,0}\delta_{L,0}, & I=1,\\ -8g'\delta(r_{ij})\delta_{S,0}\delta_{L,0}, & I=1/2,\\ 8\begin{pmatrix}g & \sqrt{2}g'\\ \sqrt{2}g' & 0\end{pmatrix}\delta(r_{ij})\delta_{S,0}\delta_{L,0}, & I=0.\end{cases} \tag{12}$$

In the above expression, S,L,I are respectively the spin, the relative orbital angular momentum and the

isospin of the system. The g and g' are dimensioned coupling constants. The Dirac delta function appearing in (12) needs to be regularized for practical calculations. As in Ref.[7, 15], we have chosen a Gaussian-like function

$$\delta(r_{ij}) \rightarrow \frac{1}{(\Lambda\sqrt{\pi})^3} \exp[-\frac{r_{ij}^2}{\Lambda^2}] \tag{13}$$

The non-central part of the OGEP consists of the tensor term $V^{\mathrm{T}}_{\mathrm{OGEP}}(\vec{r}_{ij})$ and the spin-orbit interaction $V^{\mathrm{SO}}_{\mathrm{OGEP}}(\vec{r}_{ij})$. There are several versions of tensor term in literature [26]. We use the expression derived in [8] from the QCD lagrangian in the non-relativistic limit and used subsequently by many authors (for example [28, 29])

$$V^{\mathrm{T}}_{\mathrm{OGEP}}(\vec{r}_{ij}) = -\frac{\alpha_s}{4}\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \left[\frac{1}{4M_iM_j}\frac{1}{r_{ij}^3}\right]\hat{S}_{ij} \tag{14}$$

where $\hat{S}_{ij} = 3(\vec{\sigma}_i \cdot \hat{r})(\vec{\sigma}_j \cdot \hat{r}) - \vec{\sigma}_i \cdot \vec{\sigma}_j$. The tensor potential is a scalar which is obtained by contracting two second rank tensors. Here, $\hat{r} = \hat{r}_i - \hat{r}_j$ is the unit vector in the direction of $\hat{r}$. Note that in the presence of the tensor interaction, $\vec{L}$ is no longer a good quantum number.

The spin-orbit (SO) interaction of the OGEP is given by

$$\begin{aligned} V^{\mathrm{SO}}_{\mathrm{OGEP}}(\vec{r}_{ij}) &= -\frac{\alpha_s}{4}\boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \\ &\times \left[\frac{3}{8M_iM_j}\frac{1}{r_{ij}^3}\left(\vec{r}_{ij} \times \vec{P}_{ij}\right) \cdot (\vec{\sigma}_i + \vec{\sigma}_j)\right] \end{aligned} \tag{15}$$

where the angular momentum is defined as usual in terms of relative position $\vec{r}_{ij}$ and the relative momentum $\vec{P}_{ij}$. Unlike the tensor force, the spin-orbit force does not mix states of different $\vec{L}$, since L^2 commutes with $\vec{L} \cdot \vec{S}$, $\vec{L}$ is still a constant of motion, but L_z is not.

The spin-orbit term of III is (see Refs. [7, 15]) given by

$$V^{\mathrm{SO}}_{\mathrm{III}}(\vec{r}_{ij}) = V_{\mathrm{LS}}(r_{ij})\vec{L} \cdot \vec{S} + V_{\mathrm{L}\triangle}(r_{ij})\vec{L} \cdot \vec{\triangle} \tag{16}$$

The first term in Eq. (16) is the traditional symmetric spin-orbit term proportional to the operator $\vec{L} \cdot \vec{S}$. The other term is the anti-symmetric spin-orbit term proportional to $\vec{L} \cdot \vec{\triangle}$, where $\vec{\triangle} = \frac{1}{2}(\vec{\sigma}_1 - \vec{\sigma}_2)$. The radial functions of Eq. (16) are expressed as

$$\begin{aligned} V^{\mathrm{LS}}_{\mathrm{III}}(r_{ij}) &= \left(\frac{1}{M_i^2} + \frac{1}{M_j^2}\right)\sum_{k=1}^{2}\kappa_k\frac{\exp(-r_{ij}^2/\eta_k^2)}{(\eta_k\sqrt{\pi})^3} \\ &+ \frac{1}{M_iM_j}\sum_{k=3}^{4}\kappa_k\frac{\exp(-r_{ij}^2/\eta_{k-2}^2)}{(\eta_{k-2}\sqrt{\pi})^3} \end{aligned} \tag{17}$$

and

$$V^{\mathrm{L}\Delta}_{\mathrm{III}}(r_{ij}) = \left(\frac{1}{M_i^2} - \frac{1}{M_j^2}\right)\sum_{k=5}^{6}\kappa_k\frac{\exp(-r_{ij}^2/\eta_{k-4}^2)}{(\eta_{k-4}\sqrt{\pi})^3} \tag{18}$$

The tensor term of III is

$$V^{\mathrm{T}}_{\mathrm{III}}(\vec{r}_{ij}) = \frac{\hat{S}_{ij}}{M_iM_j}\sum_{k=7}^{8}\kappa_k\frac{\exp(-r_{ij}^2/\eta_{k-4}^2)}{(\eta_{k-4}\sqrt{\pi})^3} \tag{19}$$

The term $V_{\mathrm{LS}}(r_{ij})$ is responsible for the splitting of the ${}^3L_J$ states with $J = L-1, L, L+1$. The term $V_{L\triangle}(r_{ij})$ couples states ${}^1L_{J=L}$ and ${}^3L_{J=L}$ and due to mass dependence this term is inoperative when the quarks are identical. It is to be noted that the III and the OGEP have the same spin dependence except for the $V_{L\triangle}$ term.

III. FITTING PROCEDURE

The parameters of the RHM are the masses of the quarks, $M_u = M_d$ and M_s, the respective confinement strengths, $A_u^2 = A_d^2$, A_s^2, and the oscillator size parameter b_n $(= 1/\Omega_n)$. They have been chosen to reproduce various nucleon's properties: the root mean square charge radius, the magnetic moment and the ratio of the axial coupling to the vector coupling [20]. The confinement strength $A_{u,d}$ is fixed by the stability condition for the nucleon mass against the variation of the size parameter b_n

$$\frac{\partial}{\partial b_n}\langle N\,|H|\,N\rangle = 0. \tag{20}$$

The value of b_n for S wave was 0.77 fm [24] and for P wave b_n is 0.7 fm. It is to be noted that b_n is state dependent. The parameters associated with the strange quark M_s and A_s^2 have been fitted in order to reproduce the magnetic moments of the strange baryons, according to the procedure described in [30]. The coupling constant α_s of OGEP is fixed from S wave meson spectroscopy [24]. The value of α_s turns out to be 0.2 for P wave mesons, which is compatible with the perturbative treatment. The parameters of central part of III are g, g' and Λ, the strength and the range of the interaction of III, which were fitted to the experimental masses of π and K mesons [25].

Among the non-central parts of the potential, the hyperfine terms of III has 12 additional strength and size parameters κ and η (in Eq. 17-19) respectively. We are able to reproduce the light P wave meson masses with all η and κ_1 to κ_6 parameters held fixed and by varying only the κ_7 and κ_8 parameters. The values of κ_7 and κ_8 are listed in Table II. It is to be noted that for each category of meson nonet, κ_7 is held fixed and only the κ_8 is varied. The non-central terms of OGEP are attractive, whereas the strengths of the interaction of III (i.e., κ) can have both positive and negative values [15]. We are led to the conclusion that inclusion of III in the formalism is essential. This also enables us to bring down the value of α_s to 0.2.

The $q\bar{q}$ wave function for each meson is expressed in terms of oscillator wave functions corresponding to

TABLE I: Values of parameters used in our model.

b_n	0.7 fm
$M_{\rm u,d}$	315 MeV
$M_{\rm s}$	450 MeV
α_s	0.2
$A_{\rm u}^2 = A_{\rm d}^2$	3754.7 MeV fm^{-2}
$A_{\rm s}^2$	3367.45 MeV fm^{-2}
g	0.0847 $\times$ 10^{-4} MeV^{-2}
g'	0.0535 $\times$ 10^{-4} MeV^{-2}
Λ	0.35 fm
η_1	0.194 fm
η_2	0.294 fm
η_3	0.112 fm
η_4	0.501 fm
κ_1	-2.213
κ_2	0.191
κ_3	-2.13
κ_4	2.651
κ_5	20.84
κ_6	26.43

TABLE II: Values of κ_7 and κ_8 parameters used in our model.

$N^{2S+1}L_J$	Meson	κ_7	κ_8
1^3P_0	$f_0(600)$(or σ)	-31.63	23.92
1^3P_0	$f_0(980)$	-31.63	17.35
1^3P_0	$a_0(980)$	-31.63	6.81
1^3P_0	$K_0^*(1430)$	-31.63	-21.19
1^3P_0	$a_0(1450)$	-31.63	-87.45
1^3P_0	$f_0(1500)$	-31.63	-47.26
1^3P_1	$a_1(1260)$	28.8	1.65
1^3P_1	$K_1(1270)$	28.8	-1.67
1^3P_1	$f_1(1285)$	28.8	-10.32
1^3P_1	$f_1(1420)$	28.8	18.59
1^3P_2	$f_2(1270)$	22.84	-37.95
1^3P_2	$a_2(1320)$	22.84	25.52
1^3P_2	$K_2^*(1430)$	22.84	35.19
1^3P_2	$f_2'(1525)$	22.84	47.23

the CM and relative co-ordinates. The oscillator quantum number for the CM wave functions are restricted to $N_{cm} = 0$. The Hilbert space of relative wave functions is truncated at radial quantum number $n = 2$. The Hamiltonian matrix is constructed for each meson separately in the basis states of $|N_{cm} = 0, L_{cm} = 0; N^{2S+1}L_J\rangle$ and diagonalised.

IV. RESULTS AND DISCUSSION

The masses of the singlet and triplet P-wave mesons after diagonalisation in harmonic oscillator basis with n_{max}=2 are listed in Table III and IV respectively.

Our results indicates that only the OGEP hyperfine interaction is not sufficient to reproduce the masses of the mesons. If OGEP is taken as the only source of hyperfine interaction, the value of α_s necessary to reproduce the hadrons spectrum is generally much larger than one; this leads to a large spin-orbit interaction, which destroys the overall fit to the spectrum. The inclusion of III will diminish the relative importance of OGEP for the hyperfine splittings. The important role played by the III in reproducing the masses of these mesons (as shown in Table IV) can be gauged by examining the Table V where the masses of the scalar mesons calculated after switching off the III in the full Hamiltonian are tabulated. This is because the tensor and spin-orbit terms of OGEP are attractive and hence bring down the masses of the triplet state. Also, it is important to note that, the spin-orbit terms of III are very weak [15]. The dominant contribution to the splitting of masses of axial vector mesons comes from the tensor term of III, which involves the parameters κ_7 and κ_8. It was necessary to tune κ_7 and

TABLE III: The pseudo-vector meson masses (in MeV).

Meson	$b_1(1235)$	$h_1'(1380)$	$K_1(1400)$
Experimental Mass	1229.5±3.2	1386±19	1402±7
Calculated Mass	1229.22	1386.42	1408.34

TABLE IV: The triplet meson masses (in MeV).

$N^{2S+1}L_J$	Meson	Experimental Mass	Calculated Mass
1^3P_0	$f_0(600)$(orσ)	400-1200	703.54
1^3P_0	$f_0(980)$	980 ± 10	989.72
1^3P_0	$a_0(980)$	984.7±1.2	986.84
1^3P_0	$K_0^*(1430)$	1412±6	1412.07
1^3P_0	$a_0(1450)$	1474±19	1471.77
1^3P_0	$f_0(1500)$	1507±5	1507.38
1^3P_1	$a_1(1260)$	1230±40	1229.54
1^3P_1	$K_1(1270)$	1273±7	1272.14
1^3P_1	$f_1(1285)$	1281.8±0.6	1281.15
1^3P_1	$f_1(1420)$	1426.3±0.9	1426.74
1^3P_2	$f_2(1270)$	1275.4±1.2	1276.83
1^3P_2	$a_2(1320)$	1318±0.6	1317.80
1^3P_2	$K_2^*(1430)$	1425.6±1.5	1423.61
1^3P_2	$f_2'(1525)$	1525±5	1525.9

TABLE V: The masses of scalar mesons (in MeV) after diagonalisation without III

Meson	Experimental mass	Calculated Mass
$a_0(1450)$	1474±19	1002.65
$K_0^*(1430)$	1412 ±6	1157.45
$f_0(1500)$	1507±5	1259.84

κ_8 parameters so as to get a reasonably good agreement with the experimental masses. Hence, in our model we have only two free parameters κ_7 and κ_8. Also, the results indicate that tensor part of III is crucial and plays a dominant role in explaining the masses of P wave mesons which is an important result of our investigation. The attractive or repulsive nature of III being governed by the sign of the κ. Thus by tuning the κ parameters appropriately, we are able to reproduce the meson masses in our model.

A. Pseudo-vector meson nonet (1^1P_1)

We have investigated three mesons of the 1^1P_1 pseudovector meson nonet with $J^{PC} = 1^{+-}$, namely, $b_1(1235)$, $h'_1(1380)$, $K_1(1400)$ [31]. It may be pointed out here that there is no contribution from the III for the singlet states except for the $^1P_1$ state in the K-sector. In the K-sector, the singlet P state receives a significant repulsive contribution of 129.75 MeV from the off diagonal matrix element (ME) of $\langle {}^3P_1|V_{L\Delta}|{}^1P_1\rangle$. The masses calculated by our model are tabulated against the corresponding masses [31] in Table III

B. Scalar meson nonet(1^3P_0)

The spectrum of the scalar meson nonet is very large and the actual number of resonances in the region of 1-2 GeV far exceeds the number of states which the conventional quark models can accommodate. Several of these states, however, have been interpreted as exotic mesons. It is well known that a $q\bar{q}$ meson with orbital angular momentum l and total spin s must have parity $P = (-1)^{l+1}$ and charge conjugation quantum number $C = (-1)^{l+s}$. On this basis, we define an exotic meson to be one which does not have the above spectroscopic configurations. Thus a resonance with $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}\cdots$ are exotic. Such states could be a gluonic excitation such as a hybrid ($q\bar{q}g$) or glue ball ($2g, 3g\cdots$) or a multi quark state($q\bar{q}q\bar{q}$).

The particle data group(PDG) lists isoscalar states, the $a_0(980)$ and $a_0(1450)$ [31] having masses of 984.7 ± 1.2 MeV and 1450 ± 40 MeV respectively. Theories based on chiral sigma models with three flavors [32] suggest that $a_0(980)$ would form a scalar nonet. The scalar $K_0^*(1430)$ is well established. Several groups have claimed different isoscalar structures close to 1500 MeV [33, 34]. In this work, we focus our attention only on the non-exotic scalar mesons with $J^{PC} = 0^{++}$ and assigned as $f_0(600)$, $f_0(980)$, $a_0(980)$, $K_0^*(1430)$, $a_0(1450)$ and $f_0(1500)$ [31].The isosinglet $f_0(980)$and and isotriplet $a_0(980)$ lower than 1 GeV are considered as a chiral partner (σ nonet) of the pseudo scalar ((π nonet) in SU(3) chiral symmetry.The light scalar mesons with masses less than 1 GeV are not considered as a conventional L=1 scalar nonet by many authors.

C. Axial vector meson nonet (1^3P_1)

In our model, for axial vector mesons, the tensor and $\vec{L}\cdot\vec{S}$ parts of OGEP and III have opposite signs. The contributions due to tensor terms are repulsive, whereas those due to $\vec{L}\cdot\vec{S}$ terms are attractive. As the OGEP has the same strength parameter for these terms, the contribution of the hyperfine interaction terms of OGEP is negligible whereas, due to the different strength parameters κ_i, the corresponding terms of III contribute differently. Besides, the contribution of III to the masses is also significant because of the different radial form of tensor and spin-orbit terms. We have treated κ_i as free parameters so as to reproduce the masses of $a_1(1260)$, $K_1(1270)$, $f_1(1285)$ and $f_1(1420)$. However, it should be noted that the $a_1(1260)$, with $I = 1$ has a significant width of 400 MeV and has a dominant decay channel $a_1 \rightarrow \rho\pi$. This property makes the determination of its mass difficult. The QCD sum rules [35] produce a mass of 1150 ± 40 MeV. According to Bowler [36], the a_1 mass and width are safely within the ranges $\simeq 1235\pm40$ MeV and 400 ± 100 MeV respectively. These values are in agreement with those currently adopted by PDG [31], i.e., mass of 1230 ± 40MeV and width 250 MeV to 600 MeV. In the K-sector, we have fitted to $K_1(1270)$. The contribution from the matrix element (ME) $\langle 1^1P_1|V_{L\Delta}|1^3P_1\rangle$ has been found to be significant. PDG cite two f_1 meson states [31] with $J^{PC} = 1^{++}$, namely, $f_1(1285)$ and $f_1(1420)$. There has been considerable discussion on the quark structure of these mesons [37]. We have been able to fit the masses reasonably well as shown in Table IV.

D. Tensor meson nonet (1^3P_2)

We consider some of the well established members of the tensor meson nonets, with $J^{PC} = 2^{++}$, i.e., $f_2(1270)$, $a_2(1320)$, $K_2^*(1430)$ and $f'_2(1525)$. The contributions due to tensor and $\vec{L}\cdot\vec{S}$ terms of OGEP and III bear opposite signs. The tensor potential is attractive whereas $\vec{L}\cdot\vec{S}$ part is repulsive. However, the off diagonal tensor ME $\langle 3P_2|V^{\mathrm{T}}_{OGEP}|{}^3F_2\rangle$ is strongly repulsive. In our model the mass difference between f_1 and f'_2 essentially comes from the off diagonal ME of tensor potential of OGEP and III.

In literature some more $J^{PC} = 2^{++}$ states like $f_2(1520)$, $f_2(1810)$, $f_2(2010)$, $f_2(2340)$ have been considered. Of these, $f_2(1810)$ is likely to be the 2^3P_2 state [37]. Our model prediction for $f_2(1810)$ is 1724.52 MeV.

V. CONCLUSIONS

The mass spectrum of P wave mesons is considered in the frame work of RHM with the conventional OGEP and by including III. The inclusion of III consequently diminish the relative importance of OGEP. The III also restricts α_s to be 0.2 and thus justifying the perturba-

tive truncation of multi gluon exchanges. The near mass degeneracy of the experimentally established iso-doublet states of the scalar and tensor meson nonets K_0^* and K_2^* could be accounted by the off diagonal tensor ME of OGEP and III. The simultaneous mass degeneracy of the pseudo-vector K_{1B} and axial vector K_{1A} which mix to give physical $K_1(1270)$ and $K_1(1400)$ states observed experimentally could be accounted for by the anti-symmetric spin-orbit term $V_{L\Delta}$ of III. As we have shown, RHM with OGEP and III provides a quite good description of the pseudo-vector, scalar, axial vector and tensor P-wave mesons with the same constituent quark masses, oscillator size and OGEP strength α_s. The calculated masses of the P wave mesons are in good agreement with the experimental masses.

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